

TEMPERATURE FIELDS IN OIL-GAS BEDS EXPOSED TO THE ACTION OF AN ACID

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An asymptotic solution of the problem of a temperature field in carbonate-containing oil-gas beds exposed to the action of an acid is obtained. Expressions for the concentration fields of the acid and reaction products and the porosity and temperature in the zero and first approximation have been constructed. The space-time distributions of the temperature have been calculated, and the contribution of various physical processes has been analyzed.

Under natural conditions, the skeleton of porous oil-gas beds often contains carbonate rocks. In order to increase the efficiency of production of oil or gas under such conditions, acid treatment is employed, which consists of a chemically active reagent being injected into a porous bed, which is surrounded by impermeable cap and base rocks (Fig. 1). As a result of partial corrosion of the bed skeleton, its porosity and permeability are increased. A lesser amount of specific energy is then required for oil extraction from the bed treated. The chemical reagent often used for this purpose is hydrochloric acid, which reacts with limestone ($\text{CaCO}_3 + 2\text{HCl} = \text{CaCl}_2 + \text{H}_2\text{CO}_3 + Q$). The heat of reaction Q goes for heating up the porous medium. Measurement of the magnitude of the temperature effect with the aid of thermometers sunk into a bore-hole makes it possible to control the process of acid treatment.

However, despite the wide application, the temperature fields that develop in the process of interaction of the acid with the carbonate skeleton have been inadequately studied. In this work, based on asymptotic methods, a solution was obtained for the problem of the temperature fields that develop in oil-gas beds after instantaneous injection of acid into them, when a relatively small fraction of the acid reacts for the time of injection. This regime is optimum for increasing the depth of treatment, and for this purpose special reagents that slow down the reaction rate are added into a solution [1]. In conformity with what has been said above, it is assumed for simplicity in the problem considered below that the time of acid injection into the bed is much shorter than the time of the chemical reaction in it.

Description of the Problem. We will consider a temperature problem in a cylindrical coordinate system, where the medium is represented by three regions with plane interfaces ($z_d = \pm h$) that are perpendicular to the z_d axis (Fig. 1). The first and second regions are impermeable, and the middle region of thickness $2h$ is porous. A reagent is injected into the region $-h < z_d < h$ from a bore-hole of radius r_0 , the axis of which coincides with the z_d axis. In describing the temperature problem the following assumptions were made: the porous carbonate bed is assumed to be homogeneous and anisotropic as concerns its hydrodynamic and thermophysical properties; the temperatures of the liquid and skeleton of the porous medium at each point coincide; the natural thermal field of the Earth is considered stationary (the bed is at a depth of 1–2 km; therefore, diurnal and seasonal temperature fluctuations do not reach it).

Estimates indicate that changes in the heat-conduction coefficient upon complete substitution of oil by a solution of hydrochloric acid in oil-carrying sandstone with a porosity of $m = 0.2$ do not exceed 4% with thermal conductivities not exceeding 2 W/(K·m) for the skeleton, 0.1 W/(K·m) for oil, and 0.5 W/(K·m) for water; changes in the thermal conductivity in the case of incomplete displacement or where the zone of the bed near the bore-hole is initially saturated with water are several-fold smaller. This allows one to neglect the difference in the thermal conductivity of the zone with an acid solution in the bed and the zone of displaced oil as well as the dependence of the heat-conduction coefficient on the radial coordinate, which highly simplifies the statement of the problem.

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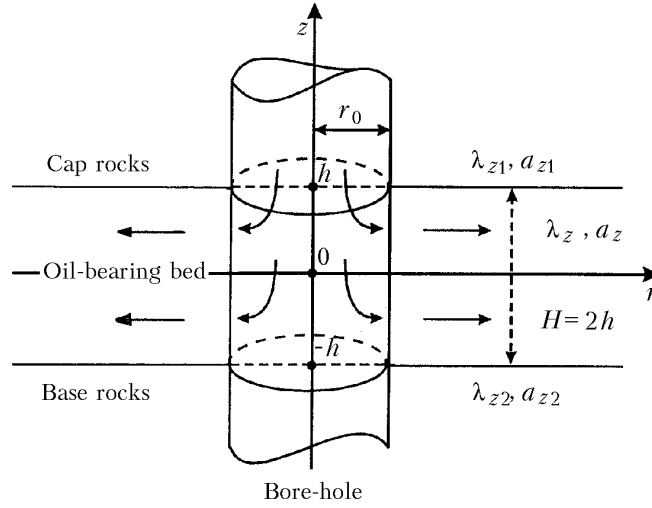


Fig. 1. Geometry of the problem.

Mathematical Statement of the Temperature Problem. For all regions the problem is represented by a heat-conduction equation (for the middle region with sources Lq_d):

$$\frac{\partial T_{1d}}{\partial t_d} = a_{r1} \frac{1}{r_d} \frac{\partial}{\partial r_d} \left(r_d \frac{\partial T_{1d}}{\partial r_d} \right) + a_{z1} \frac{\partial^2 T_{1d}}{\partial z_d^2}, \quad z_d > h, \quad r_d > 0, \quad t_d > 0; \quad (1)$$

$$\frac{\partial T_{2d}}{\partial t_d} = a_{r2} \frac{1}{r_d} \frac{\partial}{\partial r_d} \left(r_d \frac{\partial T_{2d}}{\partial r_d} \right) + a_{z2} \frac{\partial^2 T_{2d}}{\partial z_d^2}, \quad z_d < -h, \quad r_d > 0, \quad t_d > 0; \quad (2)$$

$$\frac{\partial T_d}{\partial t_d} - \frac{Lq_d}{c_{p.liq}} = a_r \frac{1}{r_d} \frac{\partial}{\partial r_d} \left(r_d \frac{\partial T_d}{\partial r_d} \right) + a_z \frac{\partial^2 T_d}{\partial z_d^2}, \quad |z_d| < h, \quad r_d > 0, \quad t_d > 0. \quad (3)$$

The conditions of equality of temperatures and heat fluxes are assigned at the boundary:

$$T_d \Big|_{z_d=h} = T_{1d} \Big|_{z_d=h}, \quad \lambda_z \frac{\partial T_d}{\partial z_d} \Big|_{z_d=h} = \lambda_{z1} \frac{\partial T_{1d}}{\partial z_d} \Big|_{z_d=h}; \quad (4)$$

$$T_d \Big|_{z_d=-h} = T_{2d} \Big|_{z_d=-h}, \quad \lambda_z \frac{\partial T_d}{\partial z_d} \Big|_{z_d=-h} = \lambda_{z2} \frac{\partial T_{2d}}{\partial z_d} \Big|_{z_d=-h}. \quad (5)$$

There are no temperature perturbations at the initial time instant:

$$T_d \Big|_{t_d=0} = T_{1d} \Big|_{t_d=0} = T_{2d} \Big|_{t_d=0} = 0. \quad (6)$$

The boundary condition is given in the form

$$\lim T_{id} \Big|_{r_d+|z_d| \rightarrow \infty} = 0. \quad (7)$$

The subscripts $i = 1$ and 2 relate to the parameters of the cap and base rocks, respectively. The solution is assumed to be limited and symmetrical on the z_d axis. An expression for the density function of the sources q_d in Eq. (3) can be found from the corresponding chemical-hydrodynamic problem.

Chemical-Hydrodynamic Problem. The mathematical statement of the problem in the zone of reaction $r_d < R$ is represented by the continuity equation [4]:

for the acid

$$\frac{d(m\rho_a)}{dt_d} = -k\alpha(m)\rho_a^l = -kq_d, \quad (8)$$

for the porous-medium skeleton

$$\frac{dm}{dt_d} = \alpha(m)\frac{\rho_a^l}{\rho_s} = \frac{q_d}{\rho_s}, \quad (9)$$

for the reaction products

$$\frac{d(m\rho_n)}{dt_d} = k_n\alpha(m)\rho_a^l, \quad (10)$$

where $i = 1, 2$ for reaction of the first and second orders, respectively; the subscript n denotes the number of the reaction product: $n = 1$ for CaCl_2 and $n = 2$ for H_2CO_3 ; $\alpha(m)$ is defined below. The initial conditions are given in the form

$$m|_{t_d=0} = m_0, \quad \rho_a|_{t_d=0} = \rho_{a0}. \quad (11)$$

The considered problem (8), (9), and (11) is nonlinear. Its solution can be found by substituting the expression for ρ_a^l from (9) into (8) with subsequent integration of the equation obtained for porosity. Finally, we arrive at the following result:

for a first-order reaction

$$\int_{m_0}^m \frac{m' dm'}{[m' - m_0(1 + \rho_{a0}/(k\rho_s))] \alpha(m')} = -kt_d, \quad (12)$$

$$\rho_a = \rho_s k \left[\frac{m_0}{m} \left(1 + \frac{\rho_{a0}}{k\rho_s} \right) - 1 \right]; \quad (13)$$

for a second-order reaction

$$\int_{m_0}^m \frac{m'^2 dm'}{[m' - m_0(1 + \rho_{a0}/(k\rho_s))]^2 \alpha(m')} = \rho_s k^2 t_d, \quad (14)$$

$$\rho_a = \rho_s k \left(\frac{m_0}{m} \left(1 + \frac{\rho_{a0}}{k\rho_s} \right) - 1 \right). \quad (15)$$

For reaction products we will have the following solution:

$$\rho_n = \frac{k_n}{m} \int_0^{t_d} \alpha(m) \rho_a^l d\tau' . \quad (16)$$

The results of the calculations performed on the basis of the above expressions are given below. Depending on the specific realization of the process, the density of sources is determined according to the expression $q_d = \alpha(m)\rho_a$ or $q_d = \alpha(m)\rho_a^2$ for chemical reactions of the first and second order, respectively.

It is known that the rate of reaction depends on the area of contact of reagents. In a porous medium this area is related to the porosity coefficient m . Below, we determine the dependences of the reaction-rate constant on porosity for the cases where a porous medium in the initial and final states can be represented as a system of pipes filled with an acid or a skeleton in the form of spheres between which there is an acid solution. For cylindrical cavities in the initial and final states the following expression was obtained:

$$\alpha(m) = \alpha_0 \sqrt{m(1-m)} ; \quad (17)$$

for cylindrical cavities in the initial state and spherical residues of the skeleton in the final state the following expression was obtained:

$$\alpha(m) = \alpha_0 \sqrt{m} \sqrt[3]{(1-m)^2} ; \quad (18)$$

for spherical cavities in the initial state and spherical residues of the skeleton in the final state the expression obtained was

$$\alpha(m) = \alpha_0 \sqrt[3]{((1-m)m)^2} , \quad (19)$$

where α_0 is the reaction-rate factor determined experimentally.

We note that the content of the acid depends on the saturability s of the acid solution in the displacement zone. The contribution of the saturability is taken into account as follows. The velocities of the liquids (and the relationship between them) are described by the Darcy equations:

$$v_o = -\frac{kf_o}{\mu_o} \frac{\partial P}{\partial r_d} , \quad v_w = -\frac{kf_w}{\mu_w} \frac{\partial P}{\partial r_d} , \quad v_o = \frac{f_o}{\mu_o} \frac{\mu_w}{f_w} v_w . \quad (20)$$

The relative permeabilities of the phases of the replaceable oil $f_o(s)$ and of the replacing aqueous solution of acid $f_w(s)$ on the saturability of this solution s are defined by the dependences

$$f_w(s) = \begin{cases} 1 , & s \geq 1 - s_{o,r} ; \\ \left(\frac{s - s_{w,r}}{1 - s_{w,r} - s_{o,r}} \right)^{3.5} , & s_{w,r} < s < 1 - s_{o,r} ; \\ 0 , & s \leq s_{w,r} ; \end{cases} \quad f_o(s) = \begin{cases} 1 , & s \leq s_{w,r} ; \\ \left(\frac{1 - s - s_{o,r}}{1 - s_{w,r} - s_{o,r}} \right)^{1.5} , & s_{w,r} < s < 1 - s_{o,r} ; \\ 0 , & s \geq 1 - s_{o,r} . \end{cases} \quad (21)$$

The value of the residual saturability with oil is approximately equal to $s_{o,r} = 0.1$ and saturability of the aqueous solution of acid to $s_{w,r} = 0.1$. We will write the continuity equations for oil and water on the assumption of incompressibility:

$$m \frac{\partial s}{\partial \tau} + \frac{1}{r_d} \frac{\partial}{\partial r_d} (r_d v_w) = 0 , \quad m \frac{\partial (1-s)}{\partial \tau} + \frac{1}{r_d} \frac{\partial}{\partial r_d} (r_d v_o) = 0 . \quad (22)$$

Here τ is the time of injection, which, by assumption, is much shorter than the time of reaction. Summing up (22) and having integrated the resulting equation, we have

$$r_d (v_w + v_o) = r_0 v_{w0}. \quad (23)$$

Having substituted Eq. (20) into Eq. (23), we find

$$v_w = \frac{r_0 v_{w0}}{r_d} \left(1 + \frac{f_o \mu_w}{f_w \mu_o} \right)^{-1}, \quad v_o = \frac{r_0 v_{w0}}{r_d} \left(1 + \frac{f_w \mu_o}{f_o \mu_w} \right)^{-1} = \frac{r_0 v_{w0}}{r_d} f(s). \quad (24)$$

Equations (24) and (22) yield the Buckley–Leverett equation for saturation:

$$\frac{\partial s}{\partial \tau} - \frac{\partial f}{\partial s} \frac{r_0 v_{w0}}{r_d m(r_d)} \frac{\partial s}{\partial r_d} = 0. \quad (25)$$

The solution of Eq. (13) under the initial condition $s(t_d = 0) = s_0(r_d)$ has the form

$$s(r_d, \tau) = s_0 \left[R^{-1} \left(\int_{r_0}^{r_d} m(r') r' dr' - F(s, \mu) \tau \right) \right], \quad F(s, \mu) = -r_0 v_{w0} \frac{\partial f}{\partial s}. \quad (26)$$

This equation is valid in the region with $r_d < R$, i.e., before the displacement front. The equation for saturation at the displacement front $r_d = R$ results from equating the saturation jump rate obtained from the mass balance in the displacement zone to the rate from relation (25):

$$\frac{\partial}{\partial s} f(s_{d,f}) = \frac{f(s_{d,f}) - f(s_o)}{s_{d,f} - s_o}. \quad (27)$$

If an aqueous solution of the acid is injected into the bed through the bore-hole, then the isosates with values of s evolve according to the relation

$$\int_{r_0}^{r_d} m(r') r' dr' = F(s, \mu) \tau. \quad (28)$$

Based on the foregoing equations, numerous calculations of saturation fields on injection of an acid solution into a bore-hole were performed. Analysis of the results of calculations has shown that at a relative viscosity $\mu = \mu_o/\mu_w$ equal to about unity (the conditions for the deposits of West Siberia) changes in saturation in the displacement zone attain $\sim 10\%$. At a relative viscosity of an order of 10, the range of change in saturation in the displacement zone is equal to about 20%. Deviations from average values of saturation under such conditions are several times smaller; therefore, in the temperature problem the change in saturation can be neglected. We note that account for the contribution of saturation presents no difficulties and is performed by formal replacement of m_i by $m_i s_i$ in the dependences given above and below.

Temperature in an Adiabatically Isolated System. The temperature field in the zone of reaction, with the thermal conductivity being ignored, is described by the equation

$$\frac{dT_d}{dt_d} = \frac{Lq_d}{c_{p,liq}}. \quad (29)$$

The volume heat capacity of the porous medium $c_{p,liq}$ is defined by the volume heat capacities of the skeleton c_s and of the saturating liquid c_{liq} , which for simplicity are assumed to be constant, $c_{p,liq} = (1 - m)c_s + mc_{liq}$. Having substituted the expression for q_d from (9) and integrated the result, we obtain the dependence of the temperature on a change in the porosity $\Delta m = m - m_0$:

$$T_d = \frac{L\rho_s}{c_{\text{liq}} - c_s} \ln \left[\frac{c_s + m(c_{\text{liq}} - c_s)}{c_s + m_0(c_{\text{liq}} - c_s)} \right] = \frac{L\rho_s}{c_{\text{liq}} - c_s} \ln \left[1 + \frac{\Delta m(c_{\text{liq}} - c_s)}{c_{\text{p.liq}}} \right] \cong \frac{L\rho_s \Delta m}{c_{\text{p.liq}}}. \quad (30)$$

The dependence of temperature on the density of the acid was obtained using Eqs. (8) and (13):

$$T_d = \frac{L\rho_s}{c_{\text{liq}} - c_s} \ln \left[\frac{c_s + m_0(c_{\text{liq}} - c_s)(k\rho_s + \rho_{a0})}{c_{\text{p.liq}}(k\rho_s + \rho_a)} \right]. \quad (31)$$

If the acid is spent entirely ($\rho_a = 0$), then the latter expression yields

$$T_d = \frac{L\rho_s}{c_{\text{liq}} - c_s} \ln \left[1 + \frac{m_0\rho_{a0}(c_{\text{liq}} - c_s)}{c_{\text{p.liq}}k\rho_s} \right] \cong \frac{m_0L\rho_{a0}}{kc_{\text{p.liq}}}. \quad (32)$$

Approximate expressions for temperature hold on condition of sufficient smallness of additions to unity under the logarithm sign.

Solution of the Temperature Problem by Asymptotic Methods. In dimensionless coordinates the problem (1)–(7) on assumption of axial symmetry takes the following form:

$$\frac{\partial T_1}{\partial t} - \frac{a_{r1}}{a_{z1}r} \frac{\partial}{\partial r} \left(r \frac{\partial T_1}{\partial r} \right) - \frac{\partial^2 T_1}{\partial z^2} = 0, \quad z > 1, \quad r > 0, \quad t > 0; \quad (33)$$

$$\frac{\partial T_2}{\partial t} - \frac{a_{r2}}{a_{z1}r} \frac{\partial}{\partial r} \left(r \frac{\partial T_2}{\partial r} \right) - \frac{a_{z2}}{a_{z1}} \frac{\partial^2 T_2}{\partial z^2} = 0, \quad z < -1, \quad r > 0, \quad t > 0; \quad (34)$$

$$\frac{\partial T}{\partial t} - q = \frac{a_r}{a_{z1}r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\chi}{\varepsilon} \frac{\partial^2 T}{\partial z^2}, \quad |z| < 1, \quad r > 0, \quad t > 0; \quad (35)$$

$$T|_{z=1} = T_1|_{z=1}, \quad \varepsilon \frac{\partial T_1}{\partial z} \Big|_{z=1} = \frac{\partial T}{\partial z} \Big|_{z=1}; \quad (36)$$

$$T|_{z=-1} = T_2|_{z=-1}, \quad \varepsilon \frac{\partial T_2}{\partial z} \Big|_{z=-1} = \frac{\lambda_{z1}}{\lambda_{z2}} \frac{\partial T}{\partial z} \Big|_{z=-1}; \quad (37)$$

$$T|_{t=0} = T_1|_{t=0} = T_2|_{t=0} = 0; \quad (38)$$

$$\lim T_i|_{r+|z| \rightarrow \infty} = 0, \quad (39)$$

where $\chi = c_1\rho_1/(c\rho)$; $r = r_d/h$; $z = z_d/h$; $t = t_d a_{z1}/h^2$; $T = T_d/T_0$; $T_i = T_{id}/T_0$; $T_0 = Lm_0\rho_{a0}/(kc_{\text{p.liq}})$; and $q = kq_d h^2/(m_0\rho_{a0}a_{z1})$.

In such a statement an analytical solution of the problem presents considerable difficulties. To obtain approximate solutions, we have employed an asymptotic method. As the expansion parameter we take the ratio of the heat conductivities of the cap medium and carbonate-containing bed $\varepsilon = \lambda_{z1}/\lambda_{z2}$. We find the solution of the problem in the form of asymptotic series in the parameter ε :

$$T = T^{(0)} + \varepsilon T^{(1)} + \varepsilon^2 T^{(2)} + \dots; \quad T_i = T_i^{(0)} + \varepsilon T_i^{(1)} + \varepsilon^2 T_i^{(2)} + \dots, \quad (40)$$

where the subscripts $i = 1, 2$ correspond to cap and base rocks, respectively, and the superscripts denote the ordinal number of the expansion coefficient. Having substituted (40) into (33)–(39) and grouped the terms with the same order in ε , we obtain

$$\sum_{j=0}^{\infty} \varepsilon^j \left(\frac{\partial T_1^{(j)}}{\partial t} - \frac{a_{r1}}{a_{z1} r} \frac{\partial}{\partial r} \left(r \frac{\partial T_1^{(j)}}{\partial r} \right) - \frac{\partial^2 T_1^{(j)}}{\partial z^2} \right) = 0, \quad z > 1, \quad r > 0, \quad t > 0; \quad (41)$$

$$\sum_{j=0}^{\infty} \varepsilon^j \left(\frac{\partial T_2^{(j)}}{\partial t} - \frac{a_{r2}}{a_{z1} r} \frac{\partial}{\partial r} \left(r \frac{\partial T_2^{(j)}}{\partial r} \right) - \frac{a_{r2}}{a_{z1}} \frac{\partial^2 T_2^{(j)}}{\partial z^2} \right) = 0, \quad z < -1, \quad r > 0, \quad t > 0; \quad (42)$$

$$-\chi \frac{\partial^2 T^{(0)}}{\partial z^2} + \sum_{j=1}^{\infty} \varepsilon^j \left(\frac{\partial T^{(j-1)}}{\partial t} - q \delta_{0,j-1} - \frac{a_r}{a_{z1} r} \frac{\partial}{\partial r} \left(r \frac{T^{(j-1)}}{r} \right) - \chi \frac{\partial^2 T^{(j)}}{\partial z^2} \right) = 0, \quad |z| < 1, \quad r > 0, \quad t > 0; \quad (43)$$

$$\sum_{j=0}^{\infty} \varepsilon^j \left(T^{(j)} \Big|_{z=1} - T_1^{(j)} \Big|_{z=1} \right) = 0; \quad (44)$$

$$\frac{\partial T^{(0)}}{\partial z} \Big|_{z=1} + \sum_{j=1}^{\infty} \varepsilon^j \left(\frac{\partial T^{(j)}}{\partial z} \Big|_{z=1} - \frac{\partial T_1^{(j-1)}}{\partial z} \Big|_{z=1} \right) = 0; \quad (45)$$

$$\frac{\lambda_{z1}}{\lambda_{z2}} \frac{\partial T^{(0)}}{\partial z} \Big|_{z=-1} + \sum_{j=1}^{\infty} \varepsilon^j \left(\frac{\lambda_{z1}}{\lambda_{z2}} \frac{\partial T^{(j)}}{\partial z} \Big|_{z=-1} - \frac{\partial T_2^{(j-1)}}{\partial z} \Big|_{z=-1} \right) = 0; \quad (46)$$

$$\sum_{j=0}^{\infty} \varepsilon^j \left(T^{(j)} \Big|_{z=-1} - T_2^{(j)} \Big|_{z=-1} \right) = 0; \quad (47)$$

$$T^{(j)} \Big|_{t=0} = 0, \quad T_i^{(j)} \Big|_{t=0} = 0, \quad \lim_{r+|z| \rightarrow \infty} T^{(j)} = 0, \quad \lim_{r+|z| \rightarrow \infty} T_i^{(j)} = 0, \quad i = 1, 2, \quad j = 1, 2, \dots \quad (48)$$

Statement of the Problem in the Zero Approximation. From (43), for the zero approximation we have $\partial^2 T^{(0)}/\partial z^2 = 0$, whence $\partial T^{(0)}/\partial z = \text{const}$. Since, according to (45) and (46), for the zero approximation the derivative with respect to z at the boundaries of the bed is equal to zero $\left(\frac{\partial T^{(0)}}{\partial z} \Big|_{z=1} = 0, \quad \frac{\partial T^{(0)}}{\partial z} \Big|_{z=-1} = 0 \right)$, it is equal to zero at all points inside the bed $\partial T^{(0)}/\partial z = 0$. The latter means that $T^{(0)}$ is independent of z and is a function of only r and t , i.e., $T^{(0)} = T^{(0)}(r, t)$. According to the first-order term in ε in (43), we find

$$\frac{\partial^2 T^{(1)}}{\partial z^2} = A(r, t) = \frac{1}{\chi} \left[\frac{\partial T^{(0)}}{\partial t} - q - \frac{a_r}{a_{z1} r} \frac{\partial}{\partial r} \left(r \frac{\partial T^{(0)}}{\partial r} \right) \right], \quad (49)$$

where $A(r, t)$ is also independent of z . Having integrated (49), we obtain an expression for the temperature in the bed in the first approximation and for its derivative with respect to z :

$$\frac{\partial T^{(1)}}{\partial z} = zA(r, t) + B(r, t), \quad (50)$$

$$T^{(1)} = \frac{z^2}{2} A(r, t) + zB(r, t) + E(r, t). \quad (51)$$

Taking into account (50) and (51) and considering the expressions for the first approximation (45) and (46), we construct a system of equations to find the unknown coefficients $A(r, t)$ and $B(r, t)$:

$$\left. \frac{\partial T^{(1)}}{\partial z} \right|_{z=1} = A(r, t) + B(r, t) = \left. \frac{\partial T_1^{(0)}}{\partial z} \right|_{z=1}, \quad (52)$$

$$\left. \frac{\partial T^{(1)}}{\partial z} \right|_{z=-1} = -A(r, t) + B(r, t) = \left. \frac{\lambda_{z2}}{\lambda_{z1}} \frac{\partial T_2^{(0)}}{\partial z} \right|_{z=-1}. \quad (53)$$

The solution of the system (52)–(53) for $A(r, t)$ and $B(r, t)$ yields

$$A(r, t) = \frac{1}{2} \left(\left. \frac{\partial T_1^{(0)}}{\partial z} \right|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}} \left. \frac{\partial T_2^{(0)}}{\partial z} \right|_{z=-1} \right), \quad B(r, t) = \frac{1}{2} \left(\left. \frac{\partial T_1^{(0)}}{\partial z} \right|_{z=1} + \frac{\lambda_{z2}}{\lambda_{z1}} \left. \frac{\partial T_2^{(0)}}{\partial z} \right|_{z=-1} \right). \quad (54)$$

With Eq. (54) taken into account, Eq. (49) is transformed as

$$\frac{\partial T^{(0)}}{\partial t} - q - \frac{a_r}{a_{z1}r} \frac{\partial}{\partial r} \left(r \frac{\partial T^{(0)}}{\partial r} \right) = \frac{\chi}{2} \left(\left. \frac{\partial T_1^{(0)}}{\partial z} \right|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}} \left. \frac{\partial T_2^{(0)}}{\partial z} \right|_{z=-1} \right). \quad (55)$$

The final statement of the temperature problem for the zero approximation has the form

$$\frac{\partial T_1^{(0)}}{\partial t} - \frac{a_{r1}}{a_{z1}r} \frac{\partial}{\partial r} \left(r \frac{\partial T_1^{(0)}}{\partial r} \right) - \frac{\partial^2 T_1^{(0)}}{\partial z^2} = 0, \quad z > 1, \quad r > 0, \quad t > 0; \quad (56)$$

$$\frac{\partial T_2^{(0)}}{\partial t} - \frac{a_{r2}}{a_{z1}r} \frac{\partial}{\partial r} \left(r \frac{\partial T_2^{(0)}}{\partial r} \right) - \frac{a_{z2}}{a_{z1}} \frac{\partial^2 T_2^{(0)}}{\partial z^2} = 0, \quad z < -1, \quad r > 0, \quad t > 0; \quad (57)$$

$$\frac{\partial T^{(0)}}{\partial t} - q - \frac{a_r}{a_{z1}r} \frac{\partial}{\partial r} \left(r \frac{\partial T^{(0)}}{\partial r} \right) = \frac{\chi}{2} \left(\left. \frac{\partial T_1^{(0)}}{\partial z} \right|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}} \left. \frac{\partial T_2^{(0)}}{\partial z} \right|_{z=-1} \right), \quad |z| < 1, \quad r > 0, \quad t > 0; \quad (58)$$

$$T^{(0)} = T_1^{(0)} \Big|_{z=1} = T_2^{(0)} \Big|_{z=-1}; \quad (59)$$

$$T^{(0)} \Big|_{t=0} = T_1^{(0)} \Big|_{t=0} = T_2^{(0)} \Big|_{t=0} = 0; \quad (60)$$

$$\lim T_i^{(0)}|_{r+|z| \rightarrow \infty} = 0. \quad (61)$$

Solution of the Problem in the Zero Approximation without Allowance for Radial Heat Conduction. Neglecting the radial heat conduction for all the regions, instead of (56)–(61) we obtain a simpler problem:

$$\frac{\partial T_1^{(0)}}{\partial t} - \frac{\partial^2 T_1^{(0)}}{\partial z^2} = 0, \quad z > 1, \quad r > 0, \quad t > 0; \quad (62)$$

$$\frac{\partial T_2^{(0)}}{\partial t} - \frac{a_{z2}}{a_{z1}} \frac{\partial^2 T_2^{(0)}}{\partial z^2} = 0, \quad z < 1, \quad r > 0, \quad t > 0; \quad (63)$$

$$\frac{\partial T^{(0)}}{\partial t} - \frac{\chi}{2} \left(\frac{\partial T_1^{(0)}}{\partial z} \Big|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}} \frac{\partial T_2^{(0)}}{\partial z} \Big|_{z=-1} \right) = q(r, t), \quad |z| < 1, \quad r > 0, \quad t > 0; \quad (64)$$

$$T^{(0)} = T_1^{(0)}|_{z=1} = T_2^{(0)}|_{z=-1}; \quad (65)$$

$$T^{(0)}|_{t=0} = T_1^{(0)}|_{t=0} = T_2^{(0)}|_{t=0} = 0; \quad (66)$$

$$\lim T_{1,2}^{(0)}|_{r+|z| \rightarrow \infty} = 0. \quad (67)$$

Having used the Laplace–Carson transformation [2]

$$T_i^{(0)\text{tr}} = p \int_0^{\infty} \exp(-pt) T_i^{(0)}(t) dt, \quad (68)$$

we write problem (62)–(67) in the space of transforms

$$pT_1^{(0)\text{tr}} - \frac{\partial^2 T_1^{(0)\text{tr}}}{\partial z^2} = 0, \quad z > 1; \quad (69)$$

$$pT_2^{(0)\text{tr}} - \frac{a_{z2}}{a_{z1}} \frac{\partial^2 T_2^{(0)\text{tr}}}{\partial z^2} = 0, \quad z < -1; \quad (70)$$

$$pT^{(0)\text{tr}} - \frac{\chi}{2} \left(\frac{\partial T_1^{(0)\text{tr}}}{\partial z} \Big|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}} \frac{\partial T_2^{(0)\text{tr}}}{\partial z} \Big|_{z=-1} \right) = q(r, p), \quad |z| < 1; \quad (71)$$

$$T^{(0)\text{tr}} = T_1^{(0)\text{tr}}|_{z=1} = T_2^{(0)\text{tr}}|_{z=-1}; \quad (72)$$

$$\lim T_{1,2}^{(0)\text{tr}}|_{r+|z| \rightarrow \infty} = 0. \quad (73)$$

Solution of problem (69)–(73) will be presented as follows:

$$T_1^{(0)\text{tr}} = T^{(0)\text{tr}} \exp(-\sqrt{p}(z-1)), \quad z > 1; \quad (74)$$

$$T_2^{(0)\text{tr}} = T^{(0)\text{tr}} \exp\left[\sqrt{p \frac{a_{z1}}{a_{z2}}}(z+1)\right], \quad z < -1. \quad (75)$$

Since

$$\left. \frac{\partial T_1^{(0)\text{tr}}}{\partial z} \right|_{z=1} = -\sqrt{p} T^{(0)\text{tr}}, \quad \left. \frac{\partial T_2^{(0)\text{tr}}}{\partial z} \right|_{z=-1} = \sqrt{p \frac{a_{z1}}{a_{z2}}} T^{(0)\text{tr}},$$

for the middle region the solution in the space of transforms has the form

$$T^{(0)\text{tr}} = \frac{q(r, p)}{p + \sqrt{p} \gamma}, \quad \gamma = \frac{\chi}{2} \left(1 + \frac{\lambda_{z2}}{\lambda_{z1}} \sqrt{\frac{a_{z1}}{a_{z2}}} \right), \quad |z| < 1. \quad (76)$$

The obtained solution (74)–(76) in the space of inverted transforms will be presented in the form

$$T^{(0)} = \int_0^t q(r, \tau') \exp(\gamma^2(t-\tau')) \operatorname{erfc}(\gamma \sqrt{t-\tau'}) d\tau', \quad |z| < 1, \quad r > 0, \quad t > 0; \quad (77)$$

$$T_1^{(0)} = \int_0^t q(r, \tau') \exp(\gamma(z-1) + \gamma^2(t-\tau')) \operatorname{erfc}\left(\frac{z-1}{2\sqrt{t-\tau'}} + \gamma \sqrt{t-\tau'}\right) d\tau', \quad z > 1, \quad r > 0, \quad t > 0; \quad (78)$$

$$T_2^{(0)} = \int_0^t q(r, \tau') \exp\left(\gamma \sqrt{\frac{a_{z1}}{a_{z2}}} |z+1| + \gamma^2(t-\tau')\right) \operatorname{erfc}\left(\frac{|z+1|}{2\sqrt{t-\tau'}} \frac{a_{z1}}{a_{z2}} + \gamma \sqrt{t-\tau'}\right) d\tau', \quad z < -1. \quad (79)$$

Solution of the Problem with Account for the Radial Component of Heat Conduction (56)–(61). In the particular case of $a_{r1} = a_{r2} = a_r$, this solution is found on the basis of the Hankel and Laplace–Carson transformations:

$$T_1^{(0)} = \frac{1}{2\Lambda} \int_0^\infty r' dr' \int_0^t \frac{q(r', \tau')}{t-\tau'} I_0\left(\frac{rr'}{2\Lambda(t-\tau')}\right) \exp\left(\gamma(z-1) + \gamma^2(t-\tau') - \frac{r^2+r'^2}{4\Lambda(t-\tau')}\right) \times \\ \times \operatorname{erfc}\left(\frac{z-1}{2\sqrt{t-\tau'}} + \gamma \sqrt{t-\tau'}\right) d\tau', \quad z > 1, \quad r > 0, \quad t > 0; \quad (80)$$

$$T_2^{(0)} = \frac{1}{2\Lambda} \int_0^\infty r' dr' \int_0^t \frac{q(r', \tau')}{t-\tau'} I_0\left(\frac{rr'}{2\Lambda(t-\tau')}\right) \exp\left(\gamma |z+1| \sqrt{\frac{a_{z1}}{a_{z2}}} + \gamma^2(t-\tau') - \frac{r^2+r'^2}{4\Lambda(t-\tau')}\right) \times$$

$$\times \operatorname{erfc} \left(\frac{|z+1|}{2\sqrt{t-\tau'}} \sqrt{\frac{a_{z1}}{a_{z2}}} + \gamma \sqrt{t-\tau'} \right) d\tau', \quad z < -1, \quad r > 0, \quad t > 0; \quad (81)$$

$$T^{(0)} = \frac{1}{2\Lambda} \int_0^\infty r' dr' \int_0^t \frac{q(r', \tau')}{t-\tau'} I_0 \left(\frac{rr'}{2\Lambda(t-\tau')} \right) \exp \left(\gamma^2(t-\tau') - \frac{r^2+r'^2}{4\Lambda(t-\tau')} \right) \operatorname{erfc} \left(\gamma \sqrt{t-\tau'} \right) d\tau', \quad |z| < 1, \quad (82)$$

where $\lambda = a_r/a_{az1}$.

Solution of the Problem in the First Approximation. To construct solutions in the first approximation and higher-order approximations, an additional integral condition is needed [3, 5]. It is obtained by averaging (35) with subsequent use of the uniqueness theorem for the solution of corresponding problems and has the form $\langle T^{(i)} \rangle = 0$. In the given problem this condition is satisfied at any values of r , as can be seen from the solutions given below. For the first coefficient of expansion, the statement of the problem, with the radial component of heat conduction being ignored, has the form

$$\frac{\partial T_1^{(1)}}{\partial t} - \frac{\partial^2 T_1^{(1)}}{\partial z^2} = 0, \quad z > 1, \quad r > 0, \quad t > 0; \quad (83)$$

$$\frac{\partial T_2^{(1)}}{\partial t} - \frac{a_{z2}}{a_{z1}} \frac{\partial^2 T_2^{(1)}}{\partial z^2} = 0, \quad z < 1, \quad r > 0, \quad t > 0; \quad (84)$$

$$\frac{\partial T^{(1)}}{\partial t} - \chi \frac{\partial^2 T^{(2)}}{\partial z^2} = 0, \quad |z| < 1, \quad r > 0, \quad t > 0; \quad (85)$$

$$\left. \frac{\partial T_1^{(1)}}{\partial z} \right|_{z=1} = \left. \frac{\partial T^{(2)}}{\partial z} \right|_{z=1}, \quad \left. \frac{\partial T_2^{(1)}}{\partial z} \right|_{z=-1} = \frac{\lambda_{z2}}{\lambda_{z1}} \left. \frac{\partial T^{(2)}}{\partial z} \right|_{z=-1}; \quad (86)$$

$$T_1^{(1)}|_{z=1} = T^{(1)}|_{z=1}, \quad T_2^{(1)}|_{z=-1} = T^{(1)}|_{z=-1}; \quad (87)$$

$$T^{(1)}|_{t=0} = T_1^{(1)}|_{t=0} = T_2^{(1)}|_{t=0} = 0; \quad (88)$$

$$\lim_{r+|z| \rightarrow \infty} T_{1,2}^{(1)} = 0. \quad (89)$$

The solution of problem (83)–(89) can be found similarly to the zero approximation. According to (85) and (51) we have

$$\hat{L}T^{(1)} = \frac{z^2}{2} \hat{L}A(r, t) + z \hat{L}B(r, t) + \hat{L}E(r, t) = \frac{\partial^2 T^{(2)}}{\partial z^2}, \quad \hat{L} = \frac{1}{\chi} \frac{\partial}{\partial t}. \quad (90)$$

Integration of the resulting expression yields

$$\frac{\partial T^{(2)}}{\partial z} = \frac{z^3}{6} \hat{L}A(r, t) + \frac{z^2}{2} \hat{L}B(r, t) + z \hat{L}E(r, t) + D(r, t). \quad (91)$$

Having compiled a system of equations for $z = 1$ and $z = -1$, we have $\hat{L}E(r, t)$:

$$\hat{L}E(r, t) = \frac{1}{2} \left(\left. \frac{\partial T_1^{(1)}}{\partial z} \right|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}} \left. \frac{\partial T_2^{(1)}}{\partial z} \right|_{z=-1} \right) - \frac{1}{6} \hat{L}A(r, t). \quad (92)$$

According to (54), (92), and (90), the equation for determining the first coefficient of expansion of $T^{(1)}$ is represented in the form

$$\begin{aligned} \hat{L}T^{(1)} = & \left(\frac{z^2}{4} - \frac{1}{12} + \frac{z}{2} \right) \hat{L} \left. \frac{\partial T_1^{(0)}}{\partial z} \right|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}} \left(\frac{z^2}{4} - \frac{1}{12} - \frac{z}{2} \right) \hat{L} \left. \frac{\partial T_2^{(0)}}{\partial z} \right|_{z=-1} + \\ & + \frac{1}{2} \left(\left. \frac{\partial T_1^{(1)}}{\partial z} \right|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}} \left. \frac{\partial T_2^{(1)}}{\partial z} \right|_{z=-1} \right), \quad |z| < 1. \end{aligned} \quad (93)$$

With account for (93), the problem (83)–(89) for the first coefficient of expansion in the space of the Laplace–Carson inverted transforms will be written in the form

$$pT_1^{(1)\text{tr}} - \frac{\partial^2 T_1^{(1)\text{tr}}}{\partial z^2} = 0, \quad z > 1; \quad (94)$$

$$pT_2^{(1)\text{tr}} - \frac{\partial^2 T_2^{(1)\text{tr}}}{\partial z^2} = 0, \quad z < -1; \quad (95)$$

$$\begin{aligned} pT^{(1)\text{tr}} - \frac{\chi}{2} \left(\left. \frac{\partial T_1^{(1)\text{tr}}}{\partial z} \right|_{z=1} - \frac{\lambda_{z2}}{\lambda_{z1}} \left. \frac{\partial T_2^{(1)\text{tr}}}{\partial z} \right|_{z=-1} \right) = & \frac{3z^2 + 6z - 1}{12} p \left. \frac{\partial T_1^{(0)\text{tr}}}{\partial z} \right|_{z=1} - \\ & - \frac{\lambda_{z2}}{\lambda_{z1}} \frac{3z^2 - 6z - 1}{12} p \left. \frac{\partial T_2^{(0)\text{tr}}}{\partial z} \right|_{z=-1}, \quad |z| < 1; \end{aligned} \quad (96)$$

$$T^{(1)\text{tr}} \Big|_{z=1} = T_1^{(1)\text{tr}} \Big|_{z=1}; \quad T^{(1)\text{tr}} \Big|_{z=-1} = T_2^{(1)\text{tr}} \Big|_{z=-1}; \quad (97)$$

$$\lim_{r+|z| \rightarrow \infty} T_{1,2}^{(1)\text{tr}} = 0. \quad (98)$$

The solution of the problem in the space of inverted transforms is determined similarly to the solution in the zero approximation:

$$T^{(1)\text{tr}} = - \frac{(3z^2 + 6z - 1 + \Omega(3z^2 - 6z - 1)) \sqrt{p} q(r, p)}{12(\sqrt{p} + \gamma)^2}, \quad \Omega = \frac{\lambda_{z2}}{\lambda_{z1}} \sqrt{\frac{a_{z1}}{a_{z2}}}, \quad |z| < 1; \quad (99)$$

$$T_1^{(1)\text{tr}} = - \frac{(2 - \Omega) \sqrt{p} q(r, p)}{3(\sqrt{p} + \gamma)^2} \exp(-\sqrt{p}(z - 1)), \quad z > 1; \quad (100)$$

$$T_2^{(1)\text{tr}} = -\frac{(2\Omega - 1)\sqrt{p} q(r, p)}{3(\sqrt{p} + \gamma)^2} \exp\left(\sqrt{p \frac{a_{z1}}{a_{z2}}}(z + 1)\right), \quad z < -1. \quad (101)$$

The solution for the first coefficients of expansion in the space of inverted transforms has the form

$$T^{(1)} = -\frac{3z^2 + 6z - 1 + \Omega(3z^2 - 6z - 1)}{12} \int_0^t q(r, \tau') \times \\ \times \left[\frac{1 + 2\gamma^2(t - \tau')}{\sqrt{\pi(t - \tau')}} - (2\gamma + 2\gamma^3(t - \tau')) \exp(\gamma^2(t - \tau')) \operatorname{erfc}(\gamma\sqrt{t - \tau'}) \right] d\tau', \quad |z| < 1, \quad r > 0, \quad t > 0; \quad (102)$$

$$T_1^{(1)} = -\frac{2 - \Omega}{3} \int_0^t q(r, \tau') \left[\frac{1 + 2\gamma^2(t - \tau')}{\sqrt{\pi(t - \tau')}} \exp\left(-\frac{(z - 1)^2}{4t}\right) - (2\gamma + 2\gamma^3(t - \tau') + \gamma^2(z - 1)) \times \right. \\ \left. \times \exp((z - 1)\gamma + \gamma^2(t - \tau')) \operatorname{erfc}\left(\frac{(z - 1)}{2\sqrt{t - \tau'}} + \gamma\sqrt{t - \tau'}\right) \right] d\tau', \quad z > 1, \quad r > 0, \quad t > 0; \quad (103)$$

$$T_2^{(1)} = -\frac{2\Omega - 1}{3} \int_0^t q(r, \tau') \left[\frac{1 + 2\gamma^2(t - \tau')}{\sqrt{\pi(t - \tau')}} \exp\left(-\frac{a_{z1}}{a_{z2}} \frac{(z + 1)^2}{4t}\right) - \left(2\gamma + 2\gamma^3(t - \tau') + \gamma^2 \sqrt{\frac{a_{z1}}{a_{z2}}} |z + 1|\right) \times \right. \\ \left. \times \exp\left(\sqrt{\frac{a_{z1}}{a_{z2}}} |z + 1| \gamma + \gamma^2(t - \tau')\right) \operatorname{erfc}\left(\sqrt{\frac{a_{z1}}{a_{z2}}} \frac{|z + 1|}{2\sqrt{t - \tau'}} + \gamma\sqrt{t - \tau'}\right) \right] d\tau', \quad z < -1, \quad r > 0, \quad t > 0. \quad (104)$$

For the first approximation, $T = T^{(0)} + \varepsilon T^{(1)}$, $T_i = T_i^{(0)} + \varepsilon T_i^{(1)}$, $i = 1, 2$. Thus, with the aid of the asymptotic method we obtained simple equations to investigate temperature fields in oil-gas beds exposed to the action of an acid.

Analysis of the Results of Calculations. Based on the solutions presented, we carried out calculations of chemical hydrodynamic and temperature fields at different coefficients of reaction $\alpha(m)$. Figure 2 presents the dependence of the density of the acid and the porosity on the dimensionless time $\alpha_0 t_d$ (t_d is the dimensional time). The calculations were performed at $\rho_s = 2930 \text{ kg/m}^3$, $\rho_{a0} = 212.5 \text{ kg/m}^3$, $m_0 = 0.15$, and $k = 0.73$. It is seen from Fig. 2 that with time the density of the injected acid is decreased and the porosity is increased. For the first-order reaction at small porosities, the rates of change in the porosity and in the acid density are minimum for the case where $\alpha(m)$ is defined by Eq. (19), corresponding to the case of spherical cavities in the initial and final states (curve 3); at the same time curves 1 and 2 practically coincide. The characteristic dimensionless time of the process is equal here to $\alpha_0 t_d \approx 3$; in the remaining cases the characteristic time of the reaction is shorter; in particular, for the second-order reaction $\alpha_0 t_d \approx 0.1$. Taking into account the influence of inhibitors, the corresponding characteristic times are increased. It follows from Fig. 2b that the change in porosity with one cycle of injection is equal approximately to 2%. As the initial porosity is increased, its corresponding increments are increased.

The foregoing shows that considerable changes in the porosity can be attained only by multiply injecting an acid. Figure 3 presents the dependence of the final porosity on the number of injections. For example, at an initial porosity of 10%, 24 cycles of acid injection are needed for the carbonate skeleton to be broken down completely.

The so-called critical porosity $m = 0.910$ is very important for practical applications. It corresponds to the case where a single injection of hydrochloric acid with a maximum density of $\rho_{a0} = 212.5 \text{ kg/m}^3$ completely attacks the carbonate bed. This means that at smaller porosities the skeleton cannot be dissolved completely as a result of a single injection of the acid.

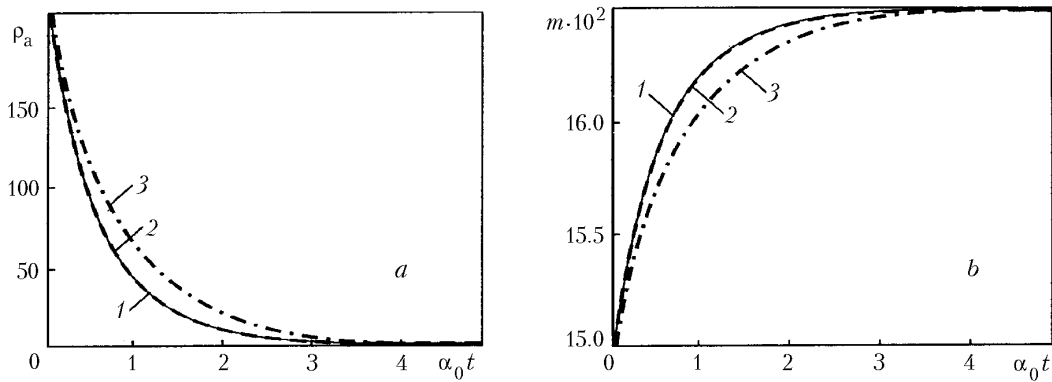


Fig. 2. Density of the acid ρ_a and porosity m vs. the time for the first-order reaction: 1) calculations by Eq. (17); 2) (18); 3) (19).

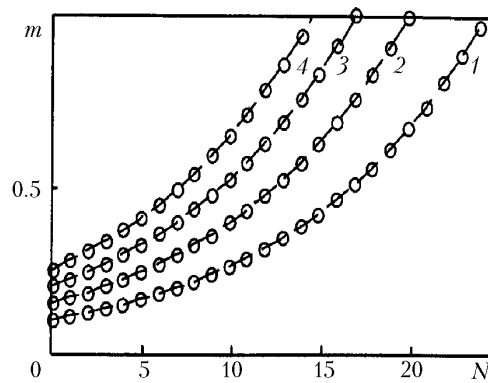


Fig. 3. Porosity m vs. the number of injections N at different initial values of the porosity m_0 : 1) $m_0 = 0.1$; 2) 0.15; 3) 0.2; 4) 0.25.

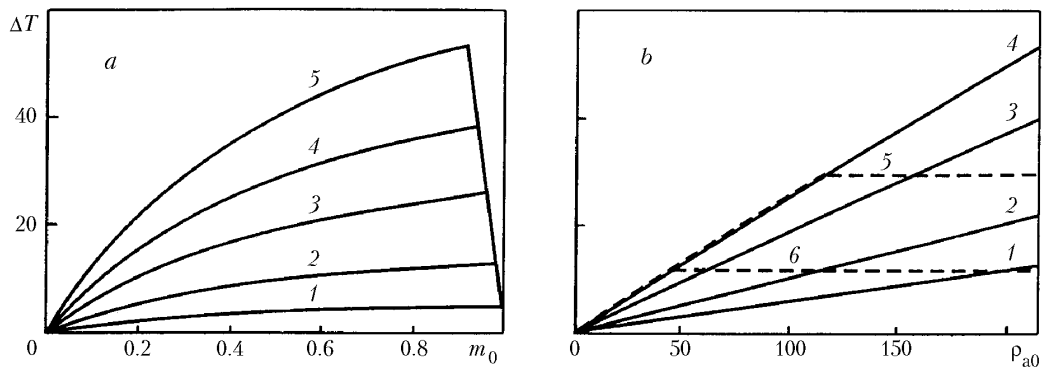


Fig. 4. Dependence of the maximum value of the thermal anomaly ΔT of a single exposure to an acid on the initial porosity m_0 (a) and on the density of the injected acid ρ_{a0} (b): (a) 1) $\rho_{a0} = 212.5$; 2) 150; 3) 100; 4) 50; 5) 20 kg/m^3 ; (b) 1) $m_0 = 0.1$; 2) 0.2; 3) 0.5; 4) 0.91; 5) 0.95; 6) 0.98.

In order to use thermal measurements in controlling acid treatment of beds it is important to know the value of the maximum temperature anomaly caused by exposure to an acid without taking into account heat exchange of the bed with the surrounding rocks (Fig. 4). From Fig. 4a it follows that the maximum value of thermal anomaly is attained at a porosity of $m = 0.91$ and a density of the injected acid in a solution of $\rho_{a0} = 212.5 \text{ kg/m}^3$ and corresponds to $\Delta T = 53.9 \text{ K}$. The calculations were carried out at $M = 0.1 \text{ kg/mole}$ (CaCO_3), $\rho_s = 2930 \text{ kg/m}^3$, $c_s = 1.67 \cdot 10^6 \text{ J/(K} \cdot \text{m}^3)$, $c_w = 4.19 \cdot 10^6 \text{ J/(K} \cdot \text{m}^3)$, $\rho_w = 1000 \text{ kg/m}^3$, and $L = 830 \text{ kJ/kg}$. The dependence of the magnitude of thermal anomaly on the density of the acid injected is linear (Fig. 4b). At an initial porosity higher than the critical one,

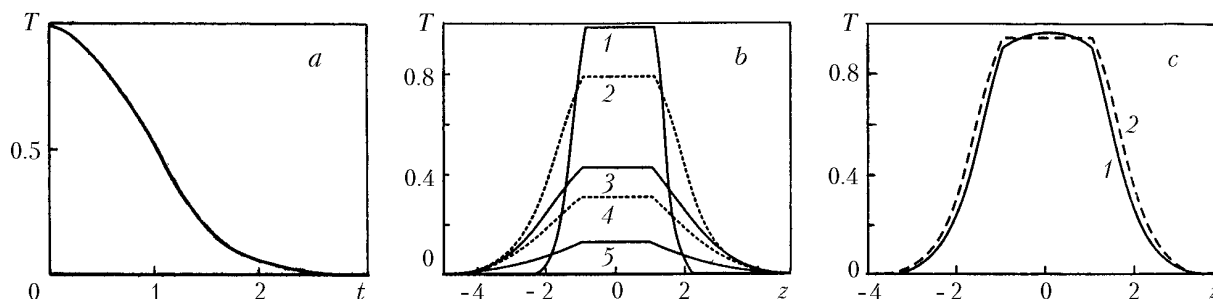


Fig. 5. Space-time distributions of the relative temperature T : a) dependence of temperature in the zero approximation in the bed on dimensionless time t (Fourier method); b) dependence of temperature in the zero approximation on the dimensionless coordinate z at different values of the dimensionless time [1] $t = 0.1$; 2) 0.5; 3) 1; 4) 1.2; 5) 1.6]; c) comparison between the first (curve 1) and zero (curve 2) approximations at $t = 0.3$.

$m > 0.91$, there are portions on the curves demonstrating that with increase in the density of the acid the temperature attains a maximum value and remains unchanged on further increase in the density; physically this corresponds to complete dissolution of the skeleton.

Space-time dependences of the temperature anomaly caused by the interaction of the acid with the skeleton on the bore-hole acid ($r = 0$) are presented in Fig. 5. The calculations were carried out at $\lambda_{z2} = \lambda_{z1}$, $a_{z1} = a_{z2}$ in the zero and first approximations. It was also assumed that the radial dimensions of the reaction zone considerably exceeded the bed thickness ($R > h$). In the opposite case, the calculations are to be performed with account for the radial heat conduction ((88)–(90)). Figure 5a depicts the dependence of the relative temperature T on the dimensionless time t (Fourier number Fo). It follows from the figure that the process of change in the temperature is completed at dimensionless times $t \approx 2$.

Figure 5b presents the dependences of the relative temperature T on the dimensionless coordinate z at different dimensionless times t in the zero approximation that corresponds to the temperature value averaged over the bed thickness. The curves in the figure make it possible to determine the size of the temperature perturbation zone, the thickness of which is approximately two times larger than that of the bed. The temperature curves in the first (curve 1) and zero (curve 2) approximations at dimensionless time $t \approx 0.3$ are compared in Fig. 5c. The comparison of these curves shows that in the zero approximation the temperature within the bed is independent of z ; the first approximation verifies the temperature distribution, since it describes in more detail its dependence on the z coordinate within the bed. It also follows from Fig. 5c that the zero approximation describes the temperature field under the indicated conditions with an accuracy sufficient for the majority of practical cases.

Thus, based on the asymptotic method, a theory has been developed that allows calculation of temperature fields in oil carbonate-containing beds exposed to the action of an acid. This opens up new prospects for developing means of controlling the process of acid effect and improving its technology.

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NOTATION

A, B, D, E, F , auxiliary functions; a_{ri}, a_{zi} , coefficients of radial and vertical thermal diffusivity of the i th layer, m^2/sec ; $c_{\text{liq}}, c_{p,\text{liq}}, c_s$, and c_w , volume heat capacity of liquid with dissolved substances, of the bed, of the skeleton material, and of water, respectively, $J/(K \cdot m^3)$; $f_o(s)$ and $f_w(s)$, phase permeability of oil and aqueous solution of acid, respectively; h , half-thickness of the bed, m ; $J_0(x)$, Bessel function of imaginary argument; $I(t)$, single Heaviside function; $J_0(x)$, Bessel function of real argument; k, k_1, k_2 , stoichiometric coefficients; L , specific heat of reaction, J/kg ; \hat{L} , operator; M , molar mass, kg/mole ; m , porosity; m_0 , initial porosity; N , number of injections of an acid solution; P , pressure, Pa ; p , parameter of the Laplace–Carson transformation; Q , reaction heat, J ; q_d and q , dimensional and dimen-

dimensionless functions of mass sources, $\text{kg}/(\text{sec}\cdot\text{m}^3)$; R , radius of the reaction zone, m; r_0 , radius of the bore-hole, m; r_d , z_d , and r , z , dimensional and dimensionless cylindrical coordinates, m; s , saturation of an aqueous solution of an acid; T_{id} , temperature of the i th zone, K; T_i , relative dimensionless temperature of the i th zone; T_d and T , dimensional and dimensionless temperatures of an oil-bearing bed, K; T_0 , maximum temperature anomaly, K; t_d and t , dimensional and dimensionless time, sec; v_o and v_w , rate of filtration of oil and of an aqueous solution of an acid, respectively, m/sec; v_{w0} , rate of filtration of an aqueous solution of an acid at the inlet, m/sec; $\alpha(m)$, reaction-rate coefficient; δ_{ij} , Kronecker delta; ε , parameter of asymptotic expansion; λ_i , thermal conductivity, W/(K·m); μ_o and μ_w , viscosity of oil and of an aqueous solution of the acid, respectively, Pa·sec; ρ_a , $\rho_{p.liq}$, ρ_s , ρ_w , ρ_1 , and ρ_2 , density of the acid in a solution, of the bed, of the skeleton material, of water, calcium chloride, and carbonic acid in a solution, respectively, kg/m^3 ; τ , time of injection, sec; χ , relative volume heat capacity; $\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-u^2) du$. Subscripts: a, acid; d, dimensional; i , ordinal number; liq, liquid; o, oil; p.liq, porous medium saturated with liquid; r, residual; r , z , directions; s, skeleton of a porous medium; w, water; d.f, displacement front; prime corresponds to integration variable. Superscript: tr, transform.

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